

Compiling CSPs: A Complexity Map of (Non-Deterministic) Multivalued Decision Diagrams

Jérôme Amilhastre¹ Hélène Fargier²
*Alexandre Niveau*³ Cédric Pralet⁴

¹Cameleon Software, Labège, France — jamilhastre@cameleon-software.com

²IRIT, Toulouse, France — fargier@irit.fr

³CRIL, Lens, France — niveau@cril.fr

⁴Onera, Toulouse, France — cpalet@onera.fr

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Introductory Example

- a product configuration problem: customized tee-shirts. [Had08]

[Had08] Tarik Hadzic, Esben Rune Hansen, and Barry O'Sullivan. "On Automata, MDDs and BDDs in Constraint Satisfaction". In: *Proceedings of the ECAI Workshop on Inference methods based on Graphical Structures of Knowledge (WIGSK)*. 2008

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 - color – black, blue, or red
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- the program must be able to tell whether each choice respects the rules

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Problem

- configurable product → **Constraint Satisfaction Problem (CSP)**

$$\left\{ \begin{array}{l} \textit{print} = \text{MiB} \quad \rightarrow \quad \textit{color} = \text{black} \\ \textit{print} = \text{StW} \quad \rightarrow \quad \textit{size} > \text{small} \\ \textit{size} \leq \text{med} \quad \vee \quad \textit{sleeves} = \text{with} \end{array} \right.$$

- each variable corresponds to a choice
- each solution is a feasible configuration

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- configuration process:
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 - is the CSP still **consistent**?
- NP-complete problem... but the user doesn't want to wait too long after each choice

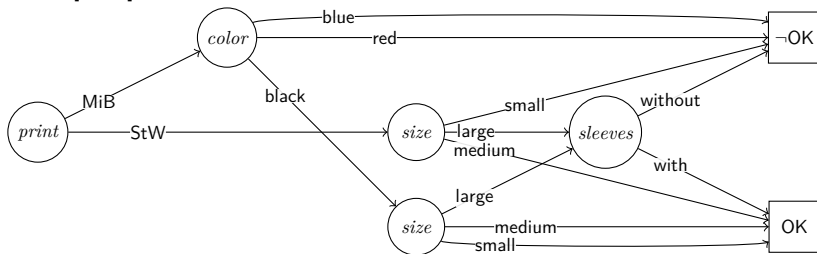
A Solution: Knowledge Compilation

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- **compile** this CSP as a Multivalued Decision Diagram (MDD)

[Sri90]

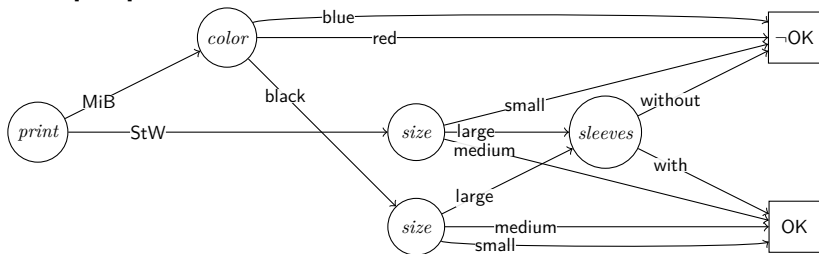


[Sri90] Arvind Srinivasan, Timothy Kam, Sharad Malik, and Robert K. Brayton. "Algorithms for Discrete Function Manipulation". In: *Proceedings of the International Conference on Computer Aided Design (ICCAD)*. Nov. 1990, pp. 92–95

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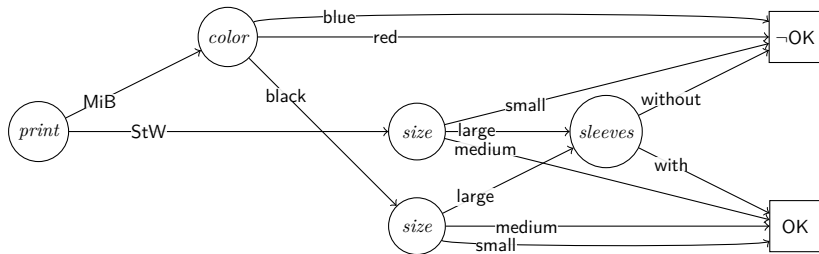
[Sri90]



- assigning values to variables (conditioning) and checking consistency are **tractable** operations on MDDs
- the user's wait is (hopefully) reduced!

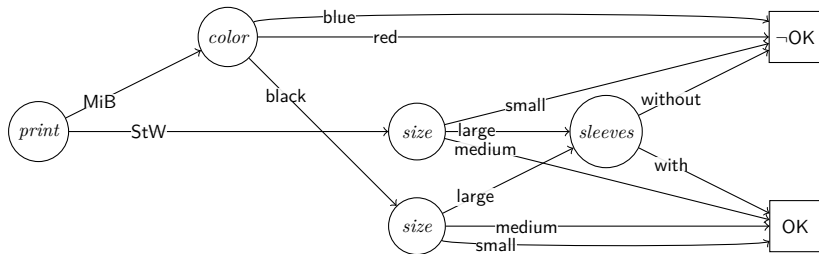
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Multivalued Decision Diagrams



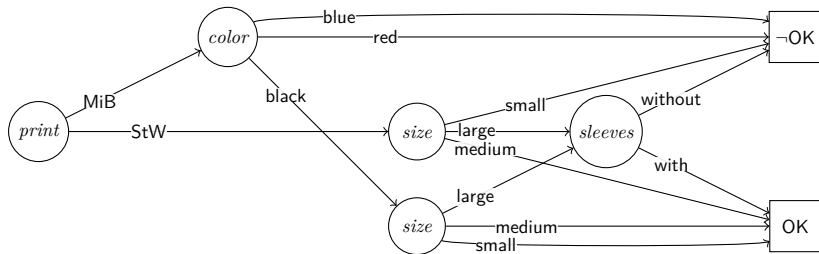
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 - determinism

Multivalued Decision Diagrams



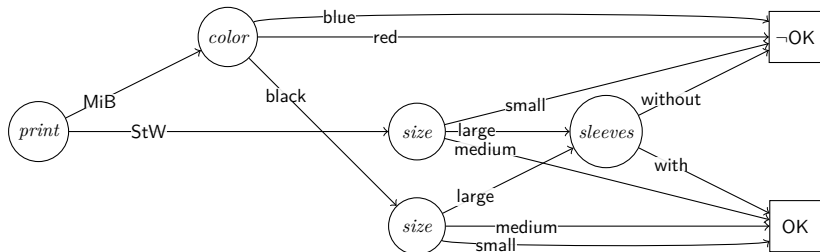
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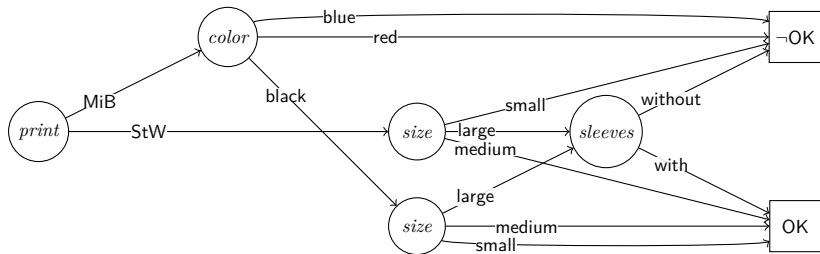


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→ they are actually “dOMDDs”

Multivalued Decision Diagrams



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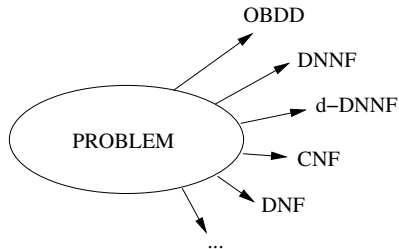
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● **our goal**: analyzing the interest of **relaxing requirements**

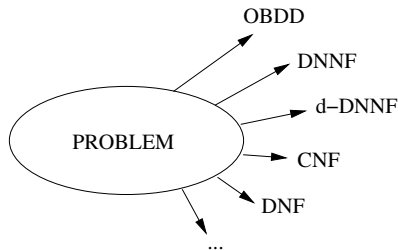
Choosing a Compilation Structure

- in the Boolean case:



Choosing a Compilation Structure

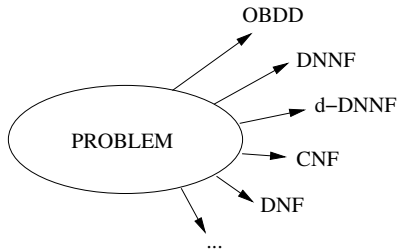
- in the Boolean case: the **knowledge compilation map** [Dar02]



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Choosing a Compilation Structure

- in the Boolean case: the **knowledge compilation map** [Dar02]



- compares structures according to two criteria:
 - ① efficiency of operations
 - ② succinctness

Knowledge Compilation Map: Operations

- all online manipulations boil down to elementary **queries** and **transformations**

L	CO (consistency)	VA (validity)	CE (clause entailmt.)	IM (implicant check)	EQ (equivalence)	SE (entailment)	CT (model count)	ME (model enum.)
NNF	○	○	○	○	○	○	○	○
DNNF	✓	○	✓	○	○	○	○	✓
BDD	○	○	○	○	○	○	○	○
FBDD	✓	✓	✓	✓	?	○	✓	✓
OBDD	✓	✓	✓	✓	✓	○	✓	✓
DNF	✓	○	✓	○	○	○	○	✓
CNF	○	✓	○	✓	○	○	○	○

L	CD (conditioning)	FO (forgetting)	SFO (single forg.)	$\wedge C$ (conjunction)	$\wedge BC$ (bounded conj.)	$\vee C$ (disjunction)	$\vee BC$ (bounded disj.)	$\neg C$ (negation)
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- ✓ polytime
- not polytime unless $P = NP$
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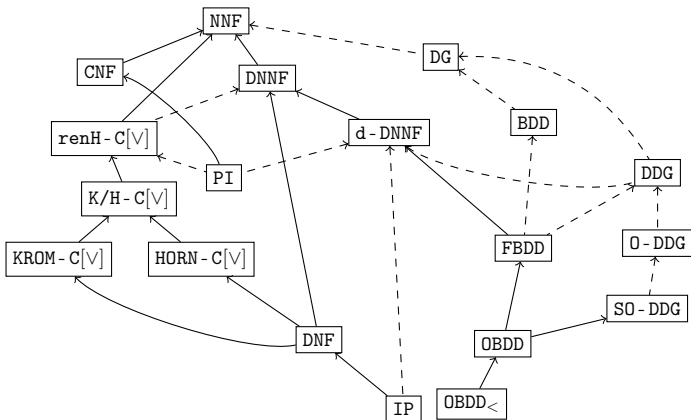
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Knowledge Compilation Map: Succinctness

- succinctness relation: orders structures w.r.t. their compacity



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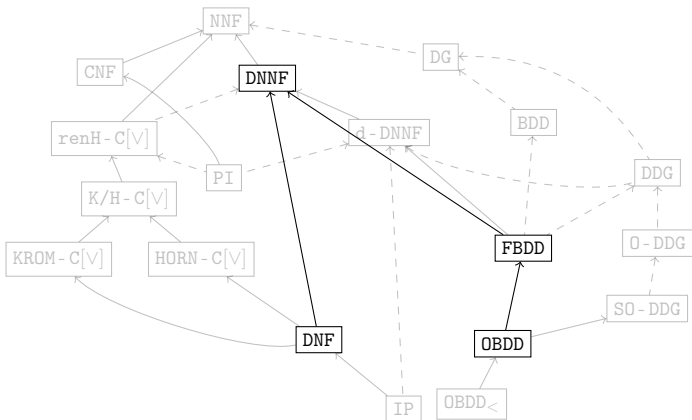
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KC Map of Deterministic MDDs

L	CO	VA	MC	CE	IM	EQ	SE	MX	CX	CT	ME
dMDD	○	○	✓	○	○	○	○	○	○	○	○
dOMDD	✓	✓	✓	✓	✓	✓	○	✓	✓	✓	✓
dOMDD _{<}	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

L	CD	TR	FO	SFO	EN	SEN	∨C	∨BC	∧C	∧BC	¬C
dMDD	✓	○	○	✓	○	✓	✓	✓	✓	✓	✓
dOMDD	✓	●	●	●	●	●	●	○	●	○	✓
dOMDD _{<}	✓	●	●	●	●	●	✓	✓	●	✓	✓

- results mainly deduced from the literature [Kam98] [Dar02] [Niv11]

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- **our work**: adding non-deterministic OMDDs to the map

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Relaxing Determinism: Operations

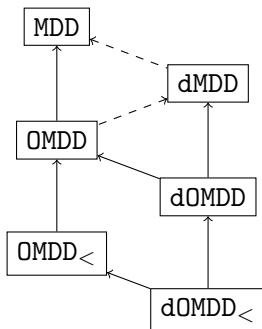
L	CO	VA	MC	CE	IM	EQ	SE	MX	CX	CT	ME
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dMDD	○	○	✓	○	○	○	○	○	○	○	○
OMDD	✓	○	✓	✓	○	○	○	✓	✓	○	✓
dOMDD	✓	✓	✓	✓	✓	○	○	✓	✓	✓	✓
OMDD _{<}	✓	○	✓	✓	○	○	○	✓	✓	○	✓
dOMDD _{<}	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

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OMDD	✓	✓	✓	✓	○	○	?	?	○	○	○
dOMDD	✓	●	●	●	●	●	●	○	●	○	✓
OMDD _{<}	✓	✓	✓	✓	○	○	✓	✓	○	✓	○
dOMDD _{<}	✓	●	●	●	●	●	●	✓	●	✓	✓

- more transformations (but loss of the negation transformation)
- fewer queries, but **CO**, **CE**, and **ME** remain

Relaxing Determinism: Succinctness

- relaxing determinism improves succinctness

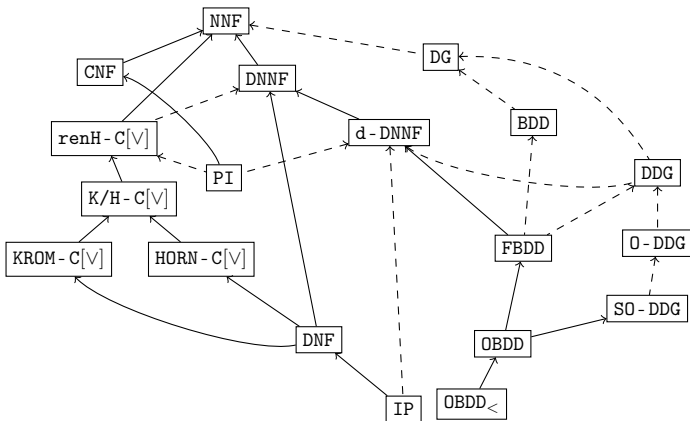


$L_1 \leftarrow L_2$: L_1 is strictly more succinct than L_2

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Relaxing Determinism: Succinctness

- interesting position in the Boolean map



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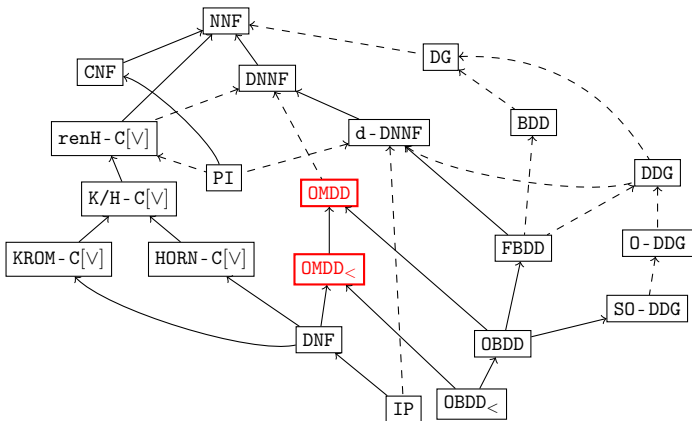
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Experiments

- compilation of randomly generated binary CSPs (15 variables, domain size 6)

%T	%C	#SOL	#N OMDD	#N dOMDD
70	10	290888073	80	81
	20	136056826	1338	1558
	30	5006576	5662	8132
	40	95131	3315	5005
	50	2367	737	897
80	10	391615179	79	80
	20	1581648506	2572	2932
	30	189551100	12223	16370
	40	11557737	20501	35486
	50	1035884	13815	25240
	60	70185	5776	9253
	70	4662	1719	2246
	80	229	54	401

Conclusion

- contributions:
 - study of the relaxation of non-determinism in decision diagrams
 - compilation map of non-deterministic MDDs
 - the $OMDD_{<}$ fragment could be successfully used in online applications such as configuration or planning
 - experimental results show practical interest
- perspectives:
 - relaxing ordering: study of non-deterministic read-once MDDs
 - adding AND nodes: non-deterministic AOMDDs
 - how to compile non-deterministic MDDs directly?