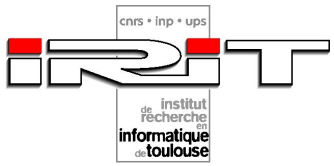
A Knowledge Compilation Map of Set-labeled Diagrams

Alexandre Niveau
Hélène Fargier Cédric Pralet



July 16th, 2011

Outline

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Introductory Example

- A product configuration problem: customized tee-shirts.
  [Hadzic et al., 2008]

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  - size – small or large
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- the program must be able to tell whether each choice respects the rules

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- configuration process:
  - each choice fixes a value for some variable → *restriction*
  - is the function still *satisfiable*? → SAT problem
- NP-complete, but the user doesn't want to wait too long after each choice (real-life configuration problems can have hundreds of multi-valued variables!...)

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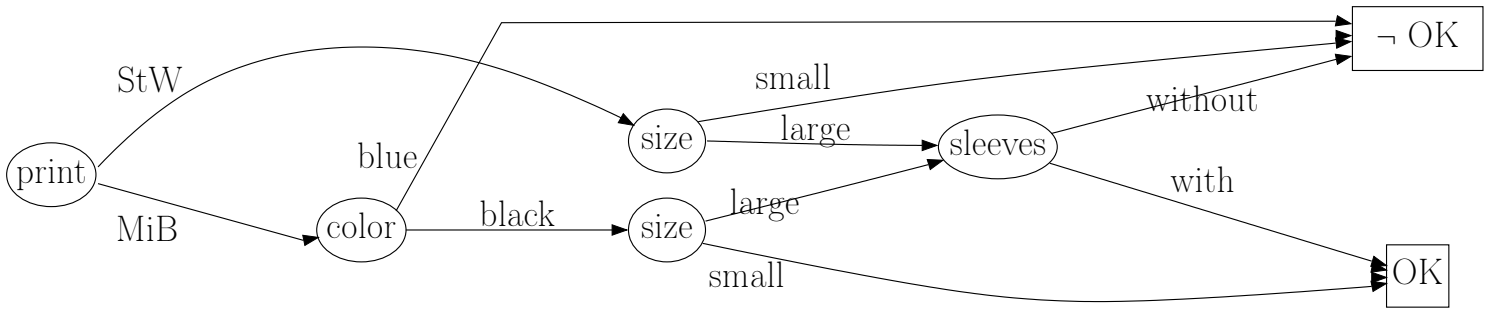

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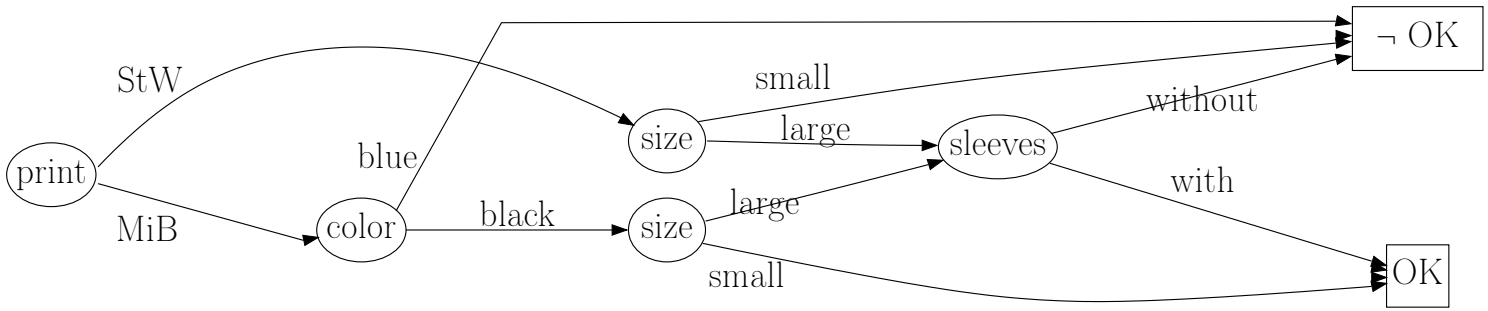
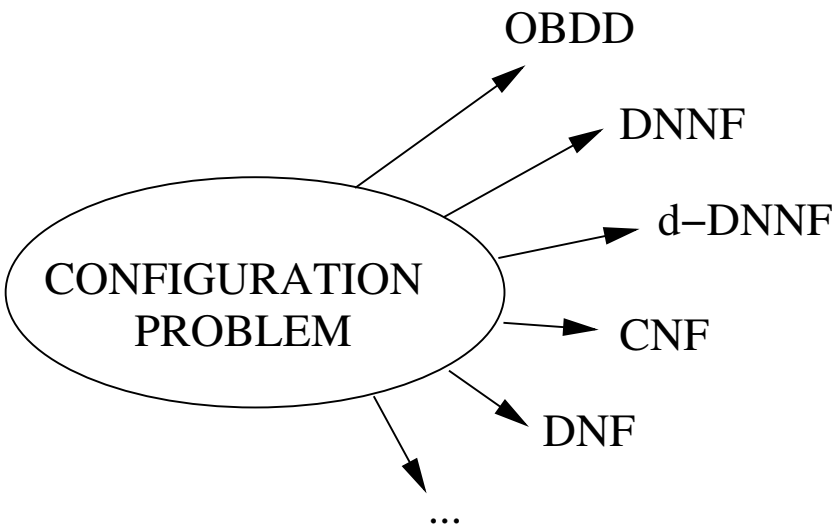


- fixing variables values (*conditioning*) and SAT are polytime on OBDDs → the user's wait is reduced!
- *knowledge compilation*: offline translating the fixed part of a problem (pre-processing, which may be hard) for online operations to be tractable

Outline

Choosing a Target Language



- what is the most appropriate for my application?

[Darwiche and Marquis, 2002] Darwiche, A. and Marquis, P. (2002). A Knowledge Compilation Map. *JAIR*, 17:229–264.

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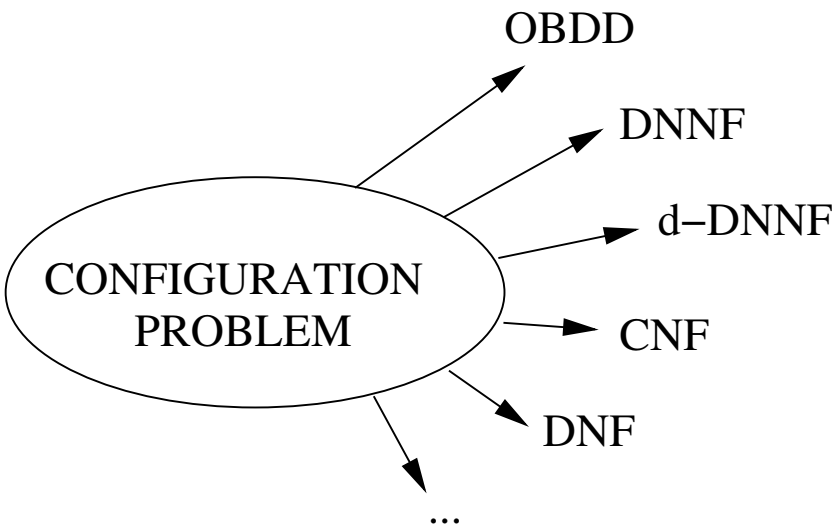

- what is the most appropriate for my application? → use the knowledge compilation map [Darwiche and Marquis, 2002]
- helps to choose languages by comparing them according to two criteria:
  - capacity to answer requests efficiently
  - succinctness.

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Knowledge Compilation Map: Requests

- all online manipulations boil down to elementary *queries* and *transformations*
- Most usual ones:
  - **CO**: consistency (does the function have a model?)
  - **VA**: validity (does the function have a countermodel?)
  - **CE**: clausal entailment (does $x = \top$ hold for all models?)
  - **CD**: conditioning (add the restriction $x = \top$ to the function)
  - **FO**: forgetting (project the function on a subset of variables)
  - $\wedge$**C**, $\vee$**C**: closure under conjunction, disjunction (compute the conjunction or disjunction of some functions)

Knowledge Compilation Map: Succinctness

- Succinctness relation: L_1 is *at least as succinct* as L_2 ($\mathrm{L}_1 \leq_s \mathrm{L}_2$) if and only if for every L_2 formula, there exists an equivalent polysize L_1 formula.

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- $\mathtt{CNF}$ and $\mathtt{DNF}$ are incomparable w.r.t. succinctness: sometimes $\mathtt{CNF}$ is better, sometimes it is $\mathtt{DNF}$

Example of Use

L	CO	VA	CE	IM	EQ	SE	CT	ME
NNF	○	○	○	○	○	○	○	○
DNNF	√	○	√	○	○	○	○	√
d-DNNF	√	√	√	√	?	○	√	√
BDD	○	○	○	○	○	○	○	○
OBDD	√	√	√	√	√	○	√	√
DNF	√	○	√	○	○	○	○	√
CNF	○	√	○	√	○	○	○	○

L	CD	FO	SFO	$\wedge$C	$\wedge$BC	$\vee$C	$\vee$BC	$\neg$C
NNF	√	○	√	√	√	√	√	√
DNNF	√	√	√	○	○	√	√	○
d-DNNF	√	○	○	○	○	○	○	?
BDD	√	○	√	√	√	√	√	√
OBDD	√	•	√	•	○	•	○	√
DNF	√	√	√	•	√	√	√	•
CNF	√	○	√	√	√	•	√	•

Example of Use

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DNNF	√	○	√	○	○	○	○	√
d-DNNF	√	√	√	√	?	○	√	√
BDD	○	○	○	○	○	○	○	○
OBDD	√	√	√	√	√	○	√	√
DNF	√	○	√	○	○	○	○	√
CNF	○	√	○	√	○	○	○	○

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BDD	√	○	√	√	√	√	√	√
OBDD	√	•	√	•	○	•	○	√
DNF	√	√	√	•	√	√	√	•
CNF	√	○	√	√	√	•	√	•

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d-DNNF	√	√	√	√	?	○	√	√
BDD	○	○	○	○	○	○	○	○
OBDD	√	√	√	√	√	○	√	√
DNF	√	○	√	○	○	○	○	√
CNF	○	√	○	√	○	○	○	○

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d-DNNF	√	○	○	○	○	○	○	?
BDD	√	○	√	√	√	√	√	√
OBDD	√	•	√	•	○	•	○	√
DNF	√	√	√	•	√	√	√	•
CNF	√	○	√	√	√	•	√	•

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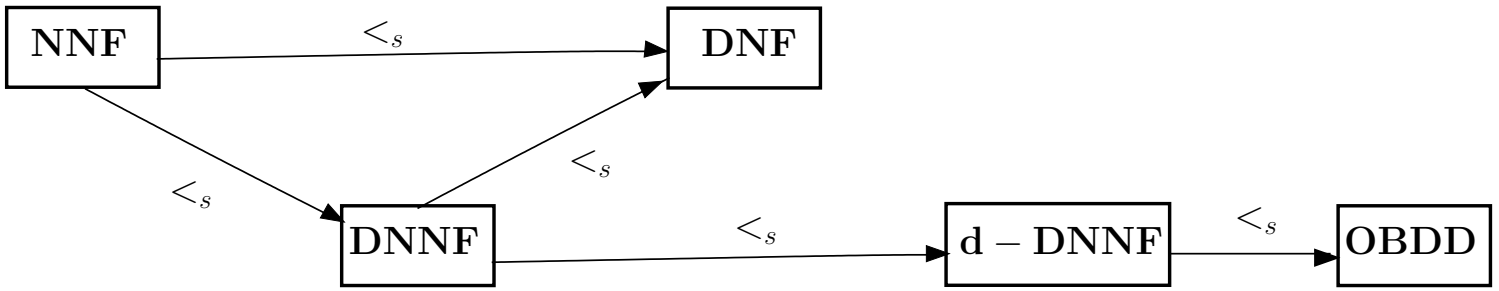
CNF	√	○	√	√	√	•	√	•



Multivalued Languages

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- the map is drawn for languages with Boolean variables only
- multivalued variables: useful *e.g.* for configuration problems
  - Multivalued Decision Diagrams (MDDs)
  - AND/OR MDDs
  - Arithmetic Circuits...
- *our work*: adding multivalued languages to the map.

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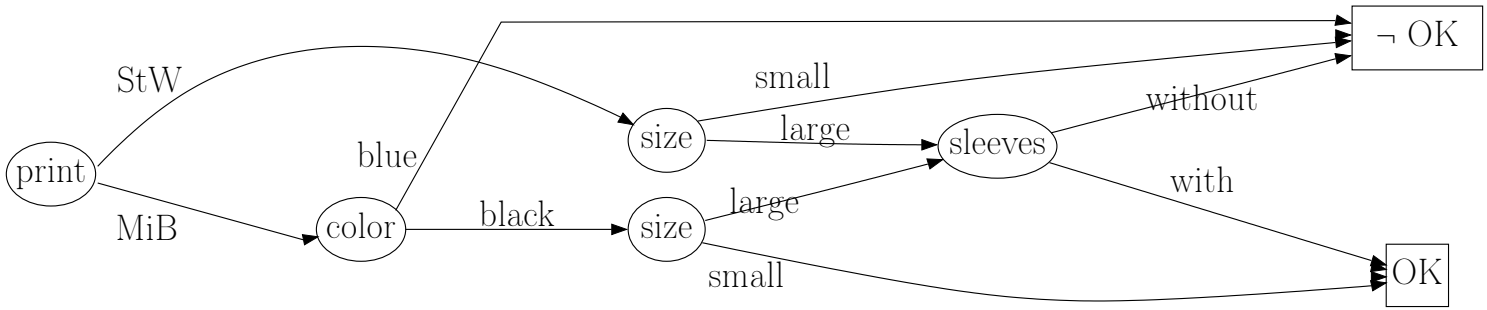
Multivalued Decision Diagrams

- MDDs: direct generalization of OBDDs to multivalued variables [Srinivasan et al., 1990]



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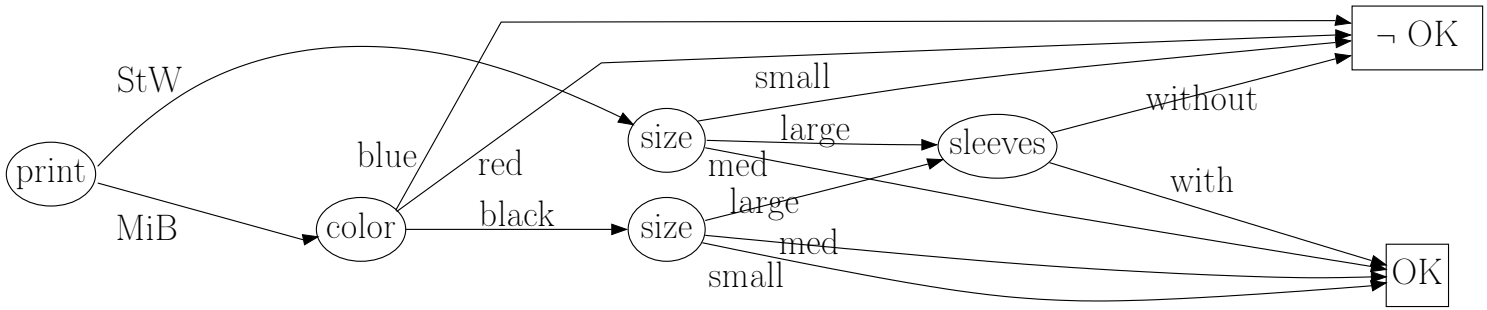

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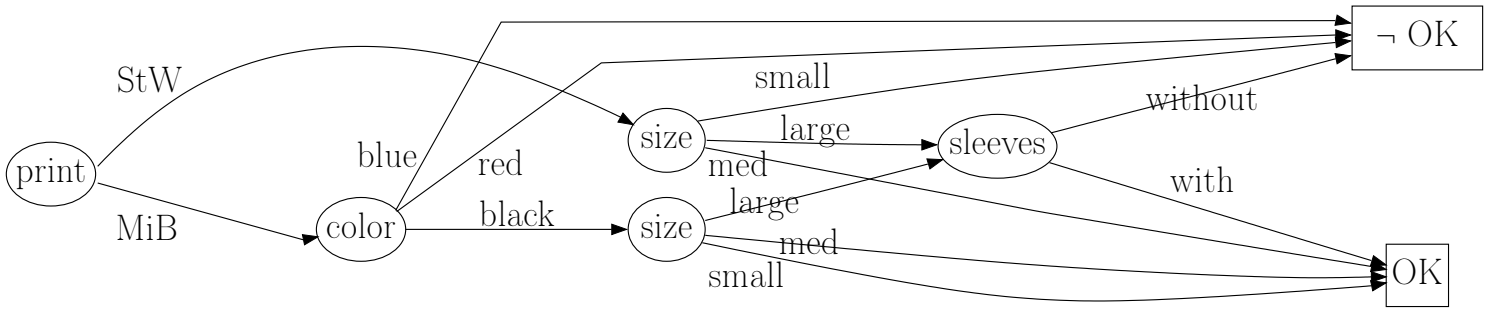

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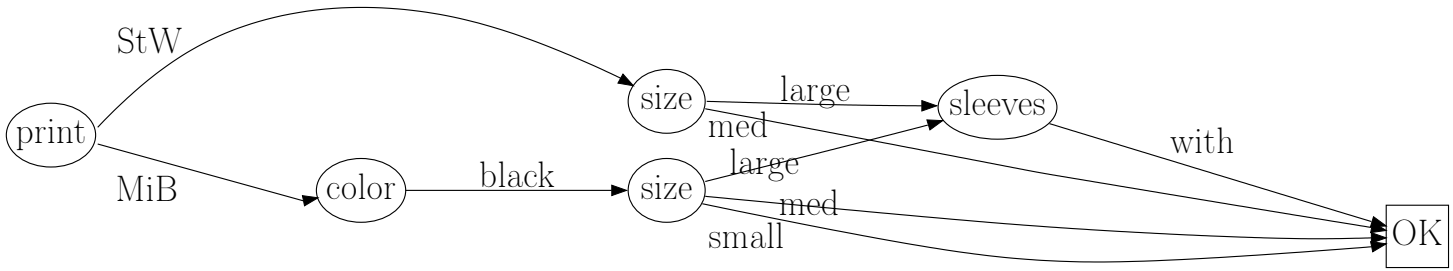

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KC Properties of MDDs

L	CO	VA	MC	CE	IM	EQ	SE	MX	CX	CT	ME
MDD	√	√	√	√	√	√	∘	√	√	√	√
$MDD_<$	√	√	√	√	√	√	√	√	√	√	√

L	CD	SCD	$\wedge$tC	FO	SFO	EN	SEN	$\wedge$C	$\wedge$BC	$\vee$C	$\vee$BC	$\top$C
MDD	√	√	√	•	•	•	•	•	∘	•	∘	√
$MDD_<$	√	√	√	•	•	•	•	•	√	•	√	√

- Results mainly found in/deduced from the literature

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L	CD	SCD	$\wedge$tC	FO	SFO	EN	SEN	$\wedge$C	$\wedge$BC	$\vee$C	$\vee$BC	$\neg$C
MDD	√	√	√	•	•	•	•	•	○	•	○	√
$MDD_<$	√	√	√	•	•	•	•	•	√	•	√	√

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- Basically the same as OBDDs except for **SFO** and **SEN**: due to domain size being unknown, the usual decomposition $\bigvee_{i \in \mathrm{Dom}(x)} \phi_{|x=i}$ gives $\phi_{|x=1} \vee \cdots \vee \phi_{|x=n}$
  $\rightarrow$ size depending on n

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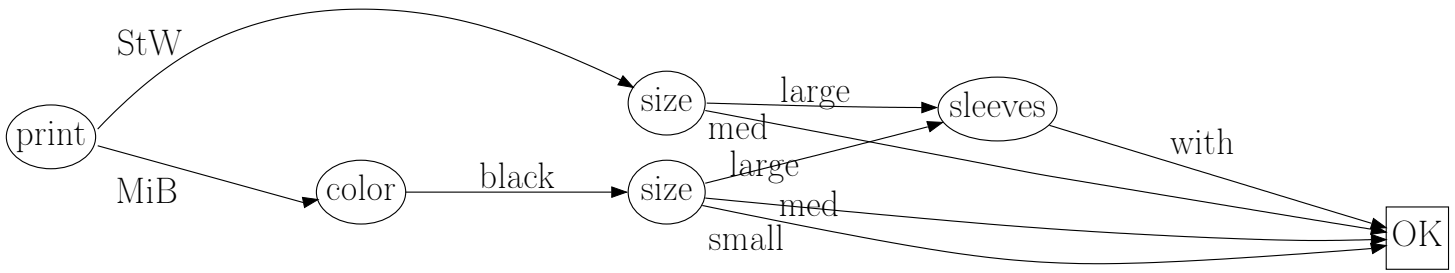

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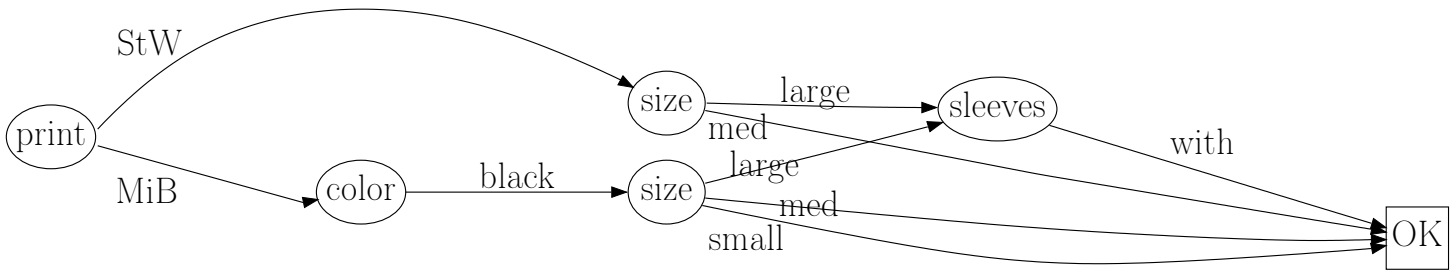

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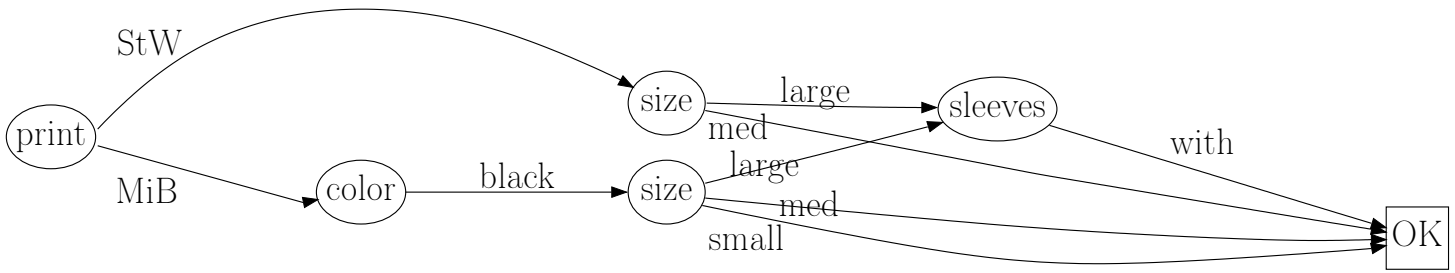

- usual definition requires ordering and determinism; we considered relaxing these constraints
- we also allow edges to be labeled by *sets* rather than single values
- we call the general language "set-labeled diagrams" (SDs)

KC Properties of SDs and dSDs

L	CO	VA	MC	CE	IM	EQ	SE	MX	CX	CT	ME
SD	○	○	√	○	○	○	○	○	○	○	○
dSD	○	○	√	○	○	○	○	○	○	○	○
MDD	√	√	√	√	√	√	○	√	√	√	√
$MDD_<$	√	√	√	√	√	√	√	√	√	√	√

L	CD	SCD	$\wedge$tC	FO	SFO	EN	SEN	$\wedge$C	$\wedge$BC	$\vee$C	$\vee$BC	$\neg$C
SD	√	√	√	○	√	○	√	√	√	√	√	?
dSD	√	√	√	○	√	○	√	√	√	√	√	√
MDD	√	√	√	•	•	•	•	•	○	•	○	√
$MDD_<$	√	√	√	•	•	•	•	•	√	•	√	√

KC Properties of SDs and dSDs

L	CO	VA	MC	CE	IM	EQ	SE	MX	CX	CT	ME
SD	○	○	√	○	○	○	○	○	○	○	○
dSD	○	○	√	○	○	○	○	○	○	○	○
MDD	√	√	√	√	√	√	○	√	√	√	√
$\text{MDD}_<$	√	√	√	√	√	√	√	√	√	√	√

L	CD	SCD	$\wedge$tC	FO	SFO	EN	SEN	$\wedge$C	$\wedge$BC	$\vee$C	$\vee$BC	$\neg$C
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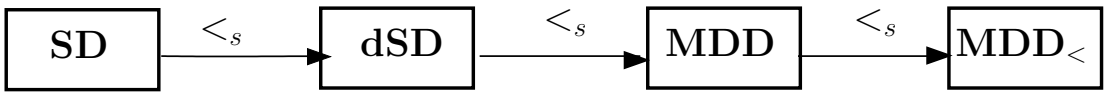
dSD	√	√	√	○	√	○	√	√	√	√	√	√
MDD	√	√	√	•	•	•	•	•	○	•	○	√
$\text{MDD}_<$	√	√	√	•	•	•	•	•	√	•	√	√



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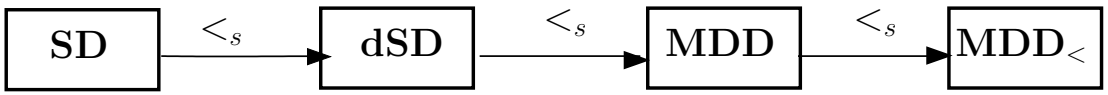
dSD	○	○	√	○	○	○	○	○	○	○	○
MDD	√	√	√	√	√	√	○	√	√	√	√
$\text{MDD}_<$	√	√	√	√	√	√	√	√	√	√	√

L	CD	SCD	$\wedge$tC	FO	SFO	EN	SEN	$\wedge$C	$\wedge$BC	$\vee$C	$\vee$BC	$\top$C
SD	√	√	√	○	√	○	√	√	√	√	√	?
dSD	√	√	√	○	√	○	√	√	√	√	√	√
MDD	√	√	√	•	•	•	•	•	○	•	○	√
$\text{MDD}_<$	√	√	√	•	•	•	•	•	√	•	√	√

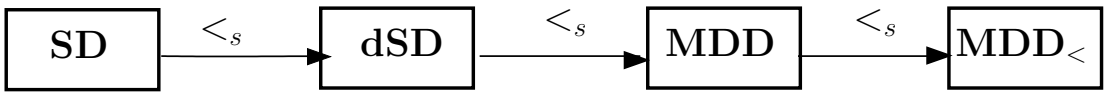


- $\text{dSD} <_s \text{MDD}$: adding ordering can lead to exponential loss in space
- does $\text{SD} <_s \text{dSD}$?

KC Properties of SDs and dSDs

L	CO	VA	MC	CE	IM	EQ	SE	MX	CX	CT	ME
SD	○	○	√	○	○	○	○	○	○	○	○
dSD	○	○	√	○	○	○	○	○	○	○	○
MDD	√	√	√	√	√	√	○	√	√	√	√
$MDD_<$	√	√	√	√	√	√	√	√	√	√	√

L	CD	SCD	$\prec$tC	FO	SFO	EN	SEN	$\prec$C	$\prec$BC	$>$C	$>$BC	$\ulcorner$C
SD	√	√	√	○	√	○	√	√	√	√	√	?
dSD	√	√	√	○	√	○	√	√	√	√	√	√
MDD	√	√	√	•	•	•	•	•	○	•	○	√
$MDD_<$	√	√	√	•	•	•	•	•	√	•	√	√



- dSD $<_s$ MDD: adding ordering can lead to exponential loss in space
- does SD $<_s$ dSD?
- neither SD nor dSD support **CO**

Focusingness

- the set labeling allows to define an interesting structural restriction allowing **CO** in polytime: *focusingness*

Focusingness

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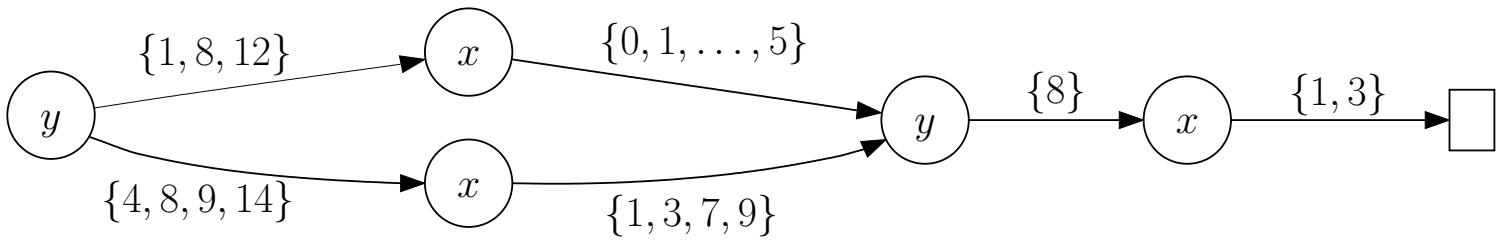
- focusingness imposes that sets pertaining to a given variable have a "nested" structure



KC Properties of FSDs and dFSDs

L	CO	VA	MC	CE	IM	EQ	SE	MX	CX	CT	ME
SD	○	○	√	○	○	○	○	○	○	○	○
dSD	○	○	√	○	○	○	○	○	○	○	○
FSD	√	○	√	√	○	○	○	√	√	○	√
dFSD	√	√	√	√	√	?	○	√	√	?	√
MDD	√	√	√	√	√	√	○	√	√	√	√
$MDD_<$	√	√	√	√	√	√	√	√	√	√	√

L	CD	SCD	$\wedge$tC	FO	SFO	EN	SEN	$<$C	$\wedge$BC	$>$C	$>$BC	$\top$C
SD	√	√	√	○	√	○	√	√	√	√	√	?
dSD	√	√	√	○	√	○	√	√	√	√	√	√
FSD	√	√	√	√	√	○	○	○	○	√	√	○
dFSD	√	√	√	○	○	○	○	○	○	○	○	?
MDD	√	√	√	•	•	•	•	•	○	•	○	√
$MDD_<$	√	√	√	•	•	•	•	•	√	•	√	√

- exact same results as DNNF and d-DNNF. However, neither FSD nor dFSD are decomposable

KC Properties of FSDs and dFSDs

L	CO	VA	MC	CE	IM	EQ	SE	MX	CX	CT	ME
SD	○	○	√	○	○	○	○	○	○	○	○
dSD	○	○	√	○	○	○	○	○	○	○	○
FSD	√	○	√	√	○	○	○	√	√	○	√
dFSD	√	√	√	√	√	?	○	√	√	?	√
MDD	√	√	√	√	√	√	○	√	√	√	√
$MDD_<$	√	√	√	√	√	√	√	√	√	√	√

L	CD	SCD	$\wedge$tC	FO	SFO	EN	SEN	$\wedge$C	$\wedge$BC	$\vee$C	$\vee$BC	$\neg$C
SD	√	√	√	○	√	○	√	√	√	√	√	?
dSD	√	√	√	○	√	○	√	√	√	√	√	√
FSD	√	√	√	√	√	○	○	○	○	√	√	○
dFSD	√	√	√	○	○	○	○	○	○	○	○	?
MDD	√	√	√	•	•	•	•	•	○	•	○	√
$MDD_<$	√	√	√	•	•	•	•	•	√	•	√	√

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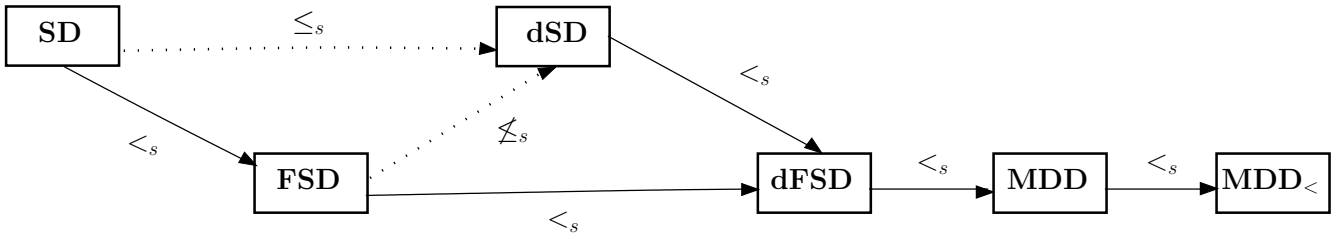

- SD $<_s$ FSD, dSD $<_s$ dFSD: imposing focusingness leads to exponential loss in space
- FSD $<_s$ dFSD: relaxing determinism on focusing structures leads to exponential gain in space

Summary

- we started to explore the compilation map of multivalued-languages
- include MDD in the map, as well as languages obtained by relaxing structural constraints: SD and dSD
- introduced languages based on focusingness

Perspectives

- almost done: explore KC properties of non-deterministic MDDs, and free MDDs/read-once SDs

  difficult remaining questions: determine whether $\mathtt{dSD} \leq_s \mathtt{SD}, \mathtt{FSD}$ and especially whether $\mathtt{FSD}$ is strictly more succinct than $\mathtt{RSD}$ (read-once SDs)

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- to do: extend the map to capture tree-ordered decision diagrams, *i.e.* AOMDDs/tree-driven automata

Thanks for your attention!