Knowledge Compilation for Online Decision-Making: Application to the Control of Autonomous Systems

Alexandre Niveau
Hélène Fargier Cédric Pralet Gérard Verfaillie

March 27th, 2012

Autonomous System

- Autonomous system: must be able to make decisions, depending on the current situation
  - current observations
  - current goals

Example (Explorer robot)

- explores a zone
- gather samples
- when enough samples have been gathered, go to another zone

→ finding a decision policy

- Problem hard to solve in the general case

Control of an Autonomous System

- Solving the problem online:
  - limited by the embedded computational power

  → reactivity not ensured

- Solving the problem offline:
  - anticipate all decisions for every possible situation
  - embed a set of “decision rules”

  → ensures a good reactivity... but limited by the embedded memory space

Looking for a Compromise

- Need for a tradeoff between reactivity and spatial compactness

$\rightarrow$ maximizing reactivity under memory space constraints

- This is the object of knowledge compilation

Principle of Knowledge Compilation

- Idea: transforming the problem into a compiled form that
  - makes its resolution tractable
  - is as compact as possible
- Can be seen as a translation of the problem into some target compilation language.
- The choice of the target language is crucial

Principle of Knowledge Compilation

- Translation step: may be hard
  - $\rightarrow$ but done offline
  - $\rightarrow$ and done only once
- Resolution step: tractable
  - $\rightarrow$ fast even online
  - $\rightarrow$ and done countless times

$\rightarrow$ Knowledge compilation shifts as much as possible of the computational effort before the system's launching

Goal of the Thesis

- There exists numerous target compilation languages:
  - the decision diagrams family (BDDs, FBDDs, OBDDs. . .);
  - finite-state automata (MDDs);
  - the NNF family (DNNFs, d-DNNFs. . .). . .

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- Proven useful in various domains:
  - model-checking
  - product configuration
  - diagnostic
  - planning

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Goal of the thesis

Study whether KC can be applied to realistic problems of aeronautical or spatial autonomous system control.

Outline

Outline

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Planning Problem

- Framework: non-deterministic planning
  - state variables, decision variables
  - initial states, goal states
  - transition relation

Example (Transition relation)

current state	**decision**	**next state**
$light$,	$press\_button$	$\neg light$, $\neg button$
$\neg light$, $\neg button$, $bulb\_OK$	$press\_button$	$light$, $button$, $bulb\_OK$
$\neg light$, $\neg button$, $bulb\_OK$	$press\_button$	$\neg light$, $button$, $\neg bulb\_OK$
$\neg bulb\_OK$	$change\_bulb$	$bulb\_OK$
...	...	...

Decision Policy

- Solution: decision policy (relation associating actions to each reachable state)

Example (Decision policy)

if you observe that...	**then...**
there is no light and the button is off	press the button
there is no light and the button is on	change the bulb
there is light	enjoy

$\rightarrow$ Boolean function δ, involving two kinds of variables:

- state variables S;
- decision variables D.

- $\delta(\vec{s}, \vec{d}) = \top \quad \rightarrow \quad \vec{d}$ is a suitable decision in state $\vec{s}$.

Decision Policy as a Boolean Function

Example (Decision policy)

if. . .	**then. . .**
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Decision Policy as a Boolean Function

Example (Decision policy)

$\vec{s}$	$\vec{d}$
$\neg light,\ \neg button$	$press\_button,\ \neg change\_bulb$
$\neg light,\ button$	$\neg press\_button,\ change\_bulb$
$light,\ button$	$\neg press\_button,\ \neg change\_bulb$
$light,\ \neg button$	$\neg press\_button,\ \neg change\_bulb$

Decision Policy as a Boolean Function

Example (Decision policy)

$\vec{s}$	$\vec{d}$	$\delta(\vec{s}, \vec{d})$
$\neg light$, $\neg button$	$press\_button$, $\neg change\_bulb$	$\top$
$\neg light$, $button$	$\neg press\_button$, $change\_bulb$	$\top$
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$light$, $\neg button$	$press\_button$, $\neg change\_bulb$	$\bot$
...	...	$\bot$
...	...	$\bot$

Exploiting a Policy

Two operations needed:

- assign state variables according to observations: conditioning

Example (Exploiting a policy)

$\vec{s}$	$\vec{d}$	$\delta(\vec{s}, \vec{d})$
$\neg light, \neg button$	$press\_button, \neg change\_bulb$	$\top$
$\neg light, button$	$\neg press\_button, change\_bulb$	$\top$
$light, button$	$\neg press\_button, \neg change\_bulb$	$\top$
$light, \neg button$	$\neg press\_button, \neg change\_bulb$	$\top$
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$light, \neg button$	$press\_button, \neg change\_bulb$	$\bot$
...	...	$\bot$

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	$\neg press\_button, \neg change\_bulb$	$\top$
	$\neg press\_button, \neg change\_bulb$	$\bot$
	$\neg press\_button, change\_bulb$	$\bot$
	$\neg press\_button, \neg change\_bulb$	$\bot$
	$press\_button, \neg change\_bulb$	$\bot$
	...	$\bot$

Exploiting a Policy

Two operations needed:

- assign state variables according to observations: conditioning
- produce one action among the possible ones: model extraction

Example (Exploiting a policy)

	$\vec{d}$	$\delta(\vec{s}, \vec{d})$
	$press\_button, \neg change\_bulb$	$\top$
	$\neg press\_button, \neg change\_bulb$	$\perp$
	$\neg press\_button, change\_bulb$	$\perp$

Exploiting a Policy

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	$\vec{d}$	$\delta(\vec{s}, \vec{d})$
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	$\neg$*press_button*, $\neg$*change_bulb*	$\bot$
	$\neg$*press_button*, *change_bulb*	$\bot$

Outline

Knowledge Compilation

- A problem is a set of operations on a knowledge base
  - knowledge base: propositional formula, constraint network, . . .
  - operations: combining formulas, checking properties, . . .
- Compilation of a problem: translation of the knowledge base into a target language such that
  - operations on the compiled form are tractable
  - the compiled form is as compact as possible

Knowledge Compilation for Planning

Applying KC to the control of autonomous systems:

- planning problem solved online
  - compilation of a transition relation
  - operations on the compiled form: conditioning, variable elimination. . .

Knowledge Compilation for Planning

Applying KC to the control of autonomous systems:

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- planning problem solved offline
  - compilation of a decision policy
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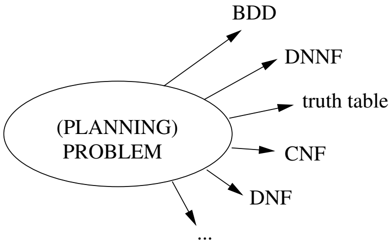
Target Languages

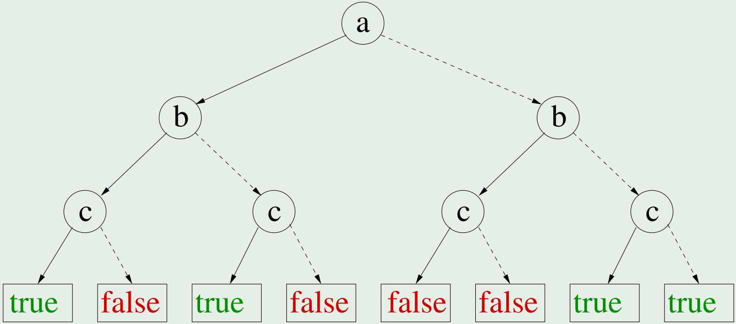
- Various target compilation languages are suitable



- An example: binary decision diagrams (BDDs)

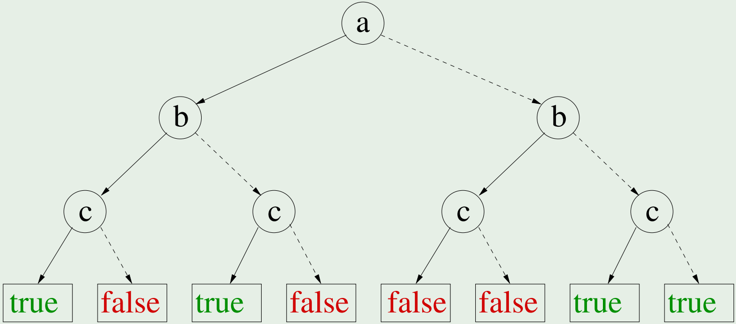
Compilation: Example of BDDs

Example : $(b \to a) \land (a \to c)$



Compilation: Example of BDDs

Example : $(b \rightarrow a) \wedge (a \rightarrow c)$



- Each path in the tree: assignment of all variables.
- The leaf is the value of the function for this assignment.

Compilation: Example of BDDs

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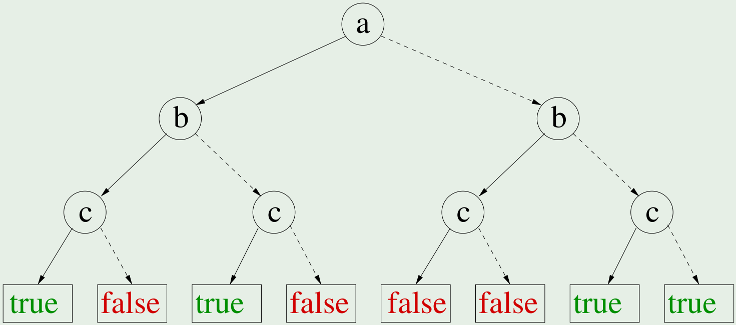
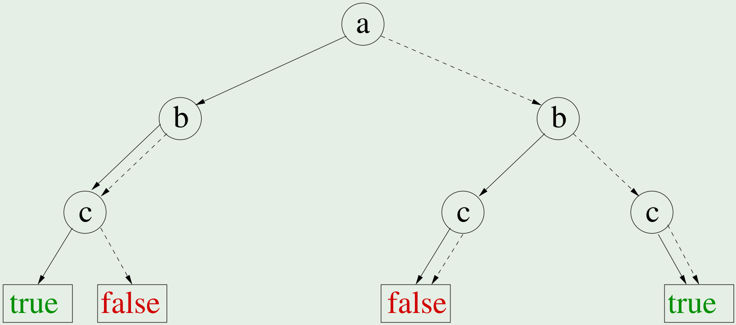


- Merging isomorphic subgraphs...

Compilation: Example of BDDs

Example : $(b \rightarrow a) \wedge (a \rightarrow c)$



- Isomorphic subgraphs merged

Compilation: Example of BDDs

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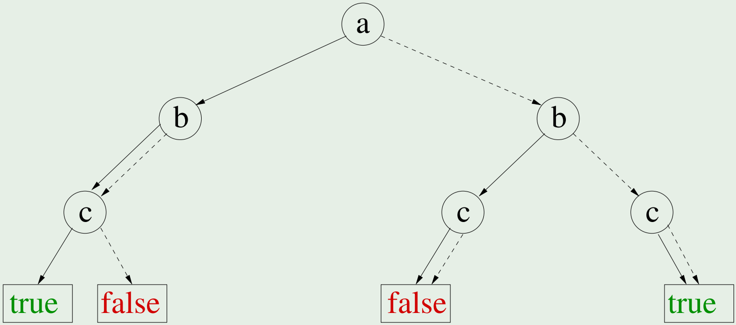
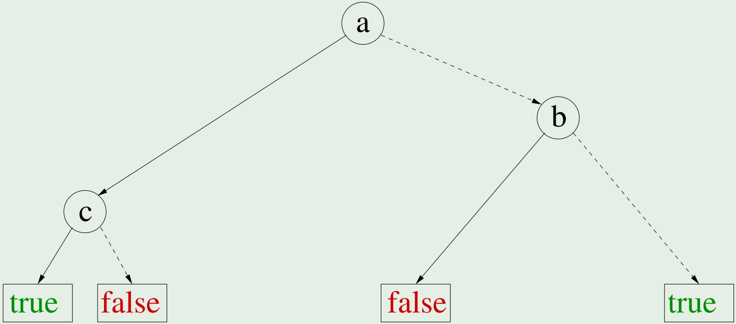


- Removing redundant nodes. . .

Compilation: Example of BDDs

Example : $(b \to a) \wedge (a \to c)$



- Redundant nodes removed

Compilation: Example of BDDs

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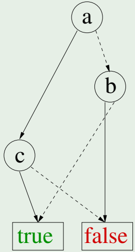
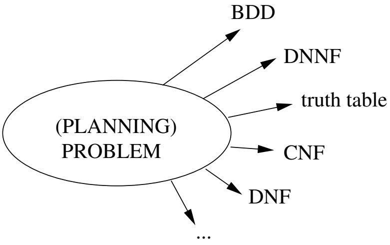


- The BDD can be exponentially more compact than the tree

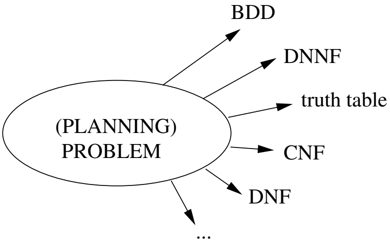
Choosing a Target Language



- What is the most appropriate for my application?

[Darwiche and Marquis, 2002] Darwiche, A. and Marquis, P. (2002). A Knowledge Compilation Map. *JAIR*, 17:229–264.

Choosing a Target Language



- What is the most appropriate for my application? → use the knowledge compilation map [Darwiche and Marquis, 2002]
- Compares target languages according to their:
  - efficiency on operations
  - succinctness.

[Darwiche and Marquis, 2002] Darwiche, A. and Marquis, P. (2002). A Knowledge Compilation Map. *JAIR*, 17:229–264.

Knowledge Compilation Map: Operations

- All online manipulations boil down to elementary queries and transformations

L	CO (consistency)	VA (validity)	MX (model extr.)	CE (clause entailmt.)	IM (implicant check)	EQ (equivalence)	SE (entailment)	CT (model count)	ME (model enum.)
NNF	○	○	○	○	○	○	○	○	○
DNNF	√	○	√	√	○	○	○	○	√
BDD	○	○	○	○	○	○	○	○	○
FBDD	√	√	√	√	√	?	○	√	√
OBDD	√	√	√	√	√	√	○	√	√
DNF	√	○	√	√	○	○	○	○	√
CNF	○	√	○	○	√	○	○	○	○

L	CD (conditioning)	FO (forgetting)	SFO (single forg.)	$\wedge$C (conjunction)	$\wedge$BC (bounded conj.)	$\vee$C (disjunction)	$\vee$BC (bounded disj.)	$\neg$C (negation)
NNF	√	○	√	√	√	√	√	√
DNNF	√	√	√	○	○	√	√	○
BDD	√	○	√	√	√	√	√	√
FBDD	√	●	○	●	○	●	○	√
OBDD	√	●	√	●	○	●	○	√
DNF	√	√	√	●	√	√	√	●
CNF	√	○	√	√	√	●	√	●

√ polytime
○ not polytime unless P = NP
● not polytime

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BDD	√	○	√	√	√	√	√	√
FBDD	√	●	○	●	○	●	○	√
OBDD	√	●	√	●	○	●	○	√
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CNF	√	○	√	√	√	●	√	●

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NNF	√	○	√	√	√	√	√	√
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BDD	√	○	√	√	√	√	√	√
FBDD	√	•	○	•	○	•	○	√
OBDD	√	•	√	•	○	•	○	√
DNF	√	√	√	•	√	√	√	•
CNF	√	○	√	√	√	•	√	•

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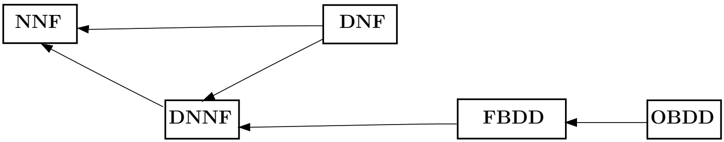
Knowledge Compilation Map: Succinctness

- Succinctness relation: orders target languages w.r.t. their compacity



- $L_1 \longleftarrow L_2$: L_1 is strictly more succinct than L_2

Target Languages for Planning Applications?

- Many target languages can be used for our application

$\rightarrow$ Boolean or enumerated variables only

- Real applications often involve continuous variables (time, energy. . .)

Our work

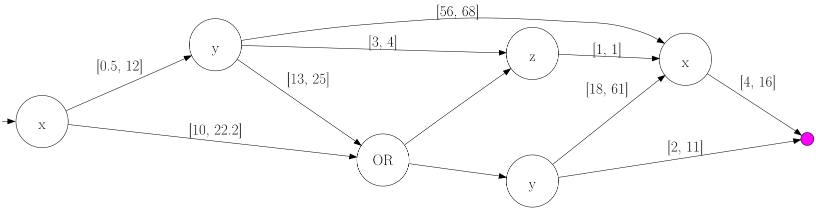
Define languages representing Boolean functions over variables with continuous or large enumerated domains.

Outline

Outline

Interval Automata

- The first language we described: interval automata (IAs).



Definition (Interval automaton)

An interval automaton (IA) is a directed acyclic graph with at most one root and at most one leaf, with nodes labeled with a variable or with "OR", and edges labeled by a closed interval from $\mathbb{R}$.

Interval Automata

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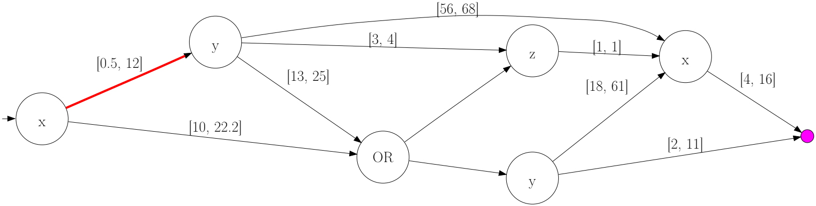

$x \in [0.5, 12]$

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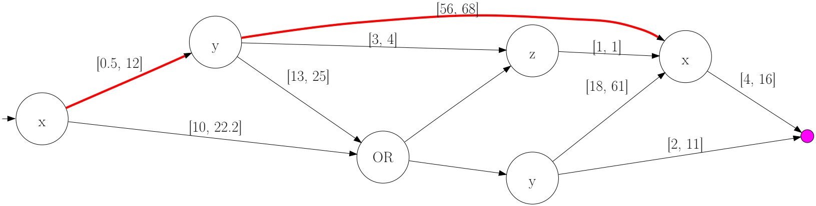

$x \in [0.5, 12] \qquad \times \qquad y \in [56, 68]$

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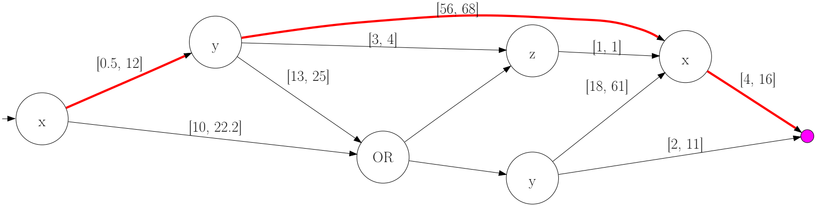


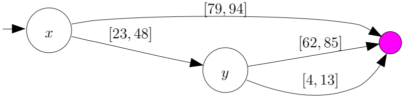
$$x \in [0.5, 12] \cap [4, 16] \quad \times \quad y \in [56, 68]$$

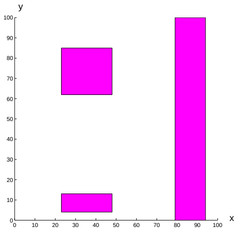
Definition (Interval automaton)

An interval automaton (IA) is a directed acyclic graph with at most one root and at most one leaf, with nodes labeled with a variable or with "OR", and edges labeled by a closed interval from $\mathbb{R}$.

Semantics of Interval Automata

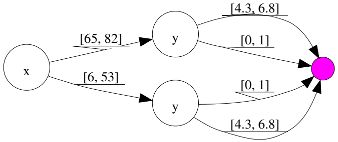
- Each interval automaton represents a Boolean function, or equivalently a set of solutions





Reduction: merging of isomorphic nodes

The size of IAs can be reduced thanks to several operations



Isomorphic nodes

Two nodes N_1, N_2 of an IA φ are isomorphic iff $\text{Var}(N_1) = \text{Var}(N_2)$ and there exists a bijection σ from $\text{Out}(N_1)$ onto $\text{Out}(N_2)$, s.t. $\forall E \in \text{Out}(N_1), \text{Lbl}(E) = \text{Lbl}(\sigma(E))$ and $\text{Dest}(E) = \text{Dest}(\sigma(E))$.

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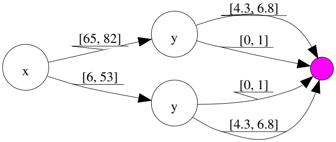
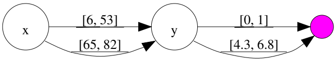


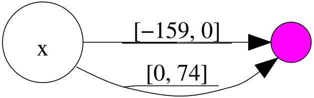
$\Downarrow$



Isomorphic nodes

Two nodes N_1, N_2 of an IA φ are isomorphic iff $\mathrm{Var}(N_1) = \mathrm{Var}(N_2)$ and there exists a bijection σ from $\mathrm{Out}(N_1)$ onto $\mathrm{Out}(N_2)$, s.t. $\forall E \in \mathrm{Out}(N_1), \mathrm{Lbl}(E) = \mathrm{Lbl}(\sigma(E))$ and $\mathrm{Dest}(E) = \mathrm{Dest}(\sigma(E))$.

Reduction: merging of contiguous edges



Contiguous edges

Two edges E_1, E_2 of an IA φ are contiguous iff $\mathrm{Src}(E_1) = \mathrm{Src}(E_2)$, $\mathrm{Dest}(E_1) = \mathrm{Dest}(E_2)$ and there exists an interval $I \subseteq \mathbb{R}$ s.t. $\mathrm{Lbl}(E_1) \cup \mathrm{Lbl}(E_2) = I \cap \mathrm{Dom}(\mathrm{Var}(E_1))$.

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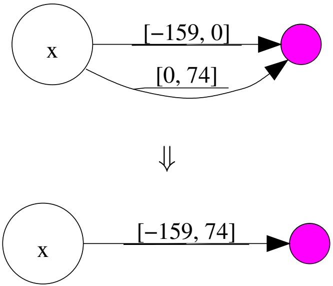

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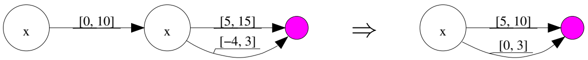
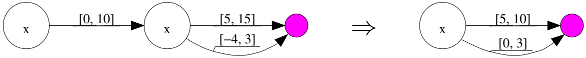
Reduction of an IA

Merging of stammering nodes:

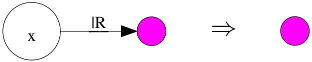


Reduction of an IA

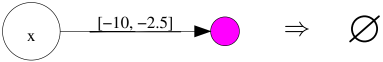
Merging of stammering nodes:



Elimination of undecisive nodes:



Elimination of unreachable edges (here $\mathrm{Dom}(x) = \mathbb{R}_+$):



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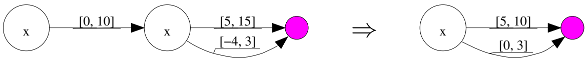

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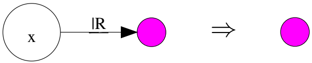

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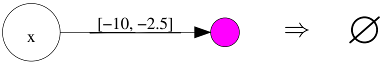
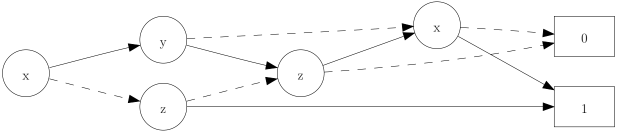


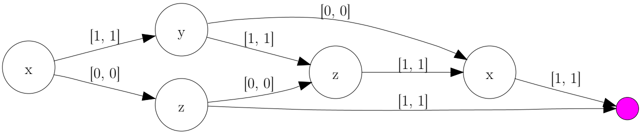
Theorem (Reduction of an IA)

There exists a polytime algorithm transforming any IA into an equivalent reduced IA.

Relation with BDD

- BDDs are particular IAs (Boolean variables, deterministic nodes).



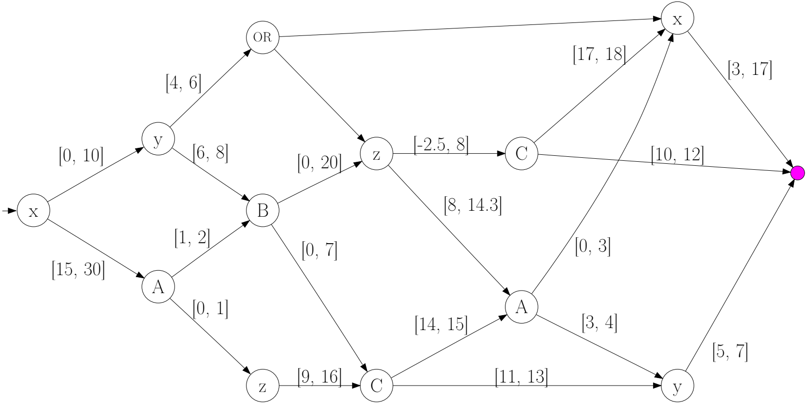


Outline

Conditioning an IA

We observe the state $x = 17, y = 6, z = 8$.

$\longrightarrow$ Conditioning of the policy :



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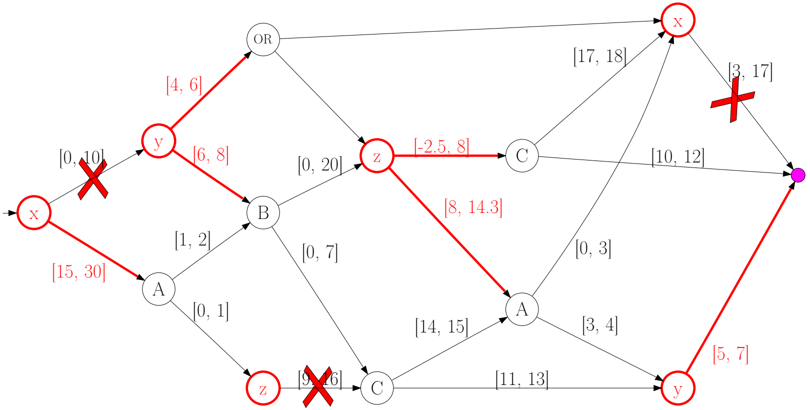

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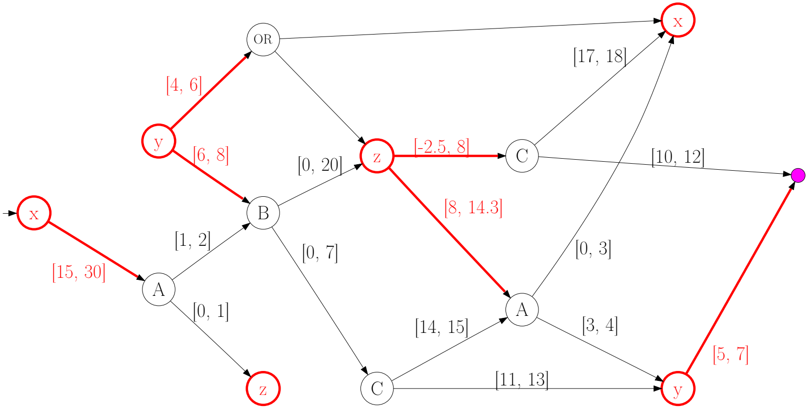

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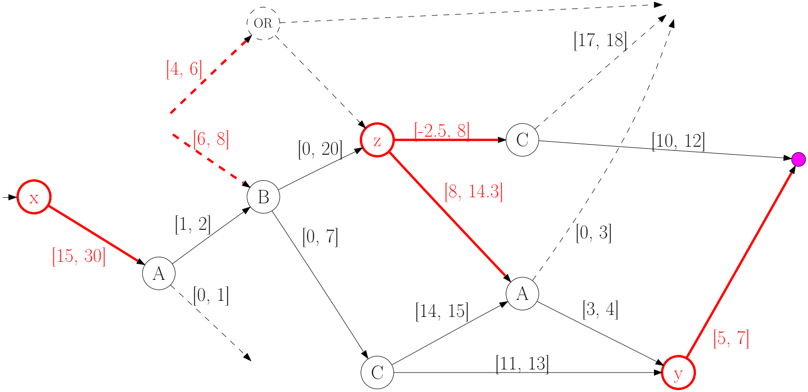

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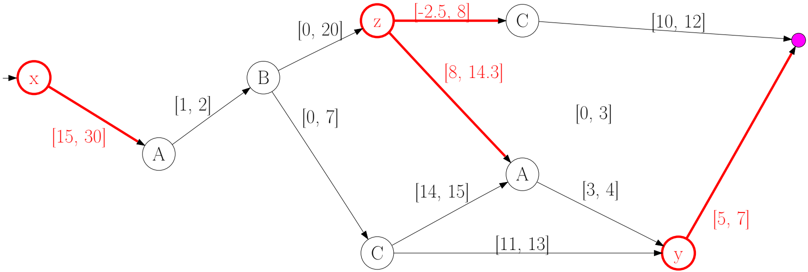

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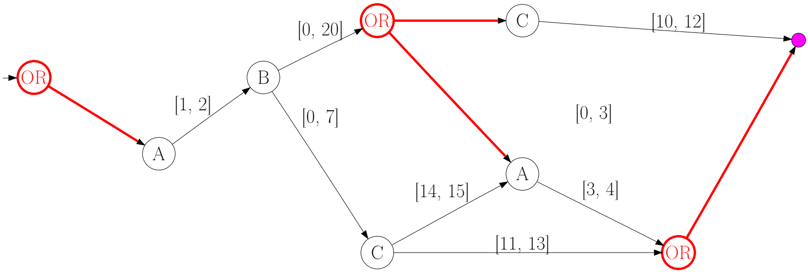
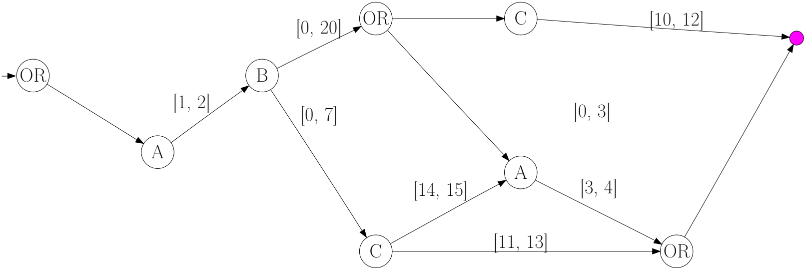


Model Extraction on an IA

We obtain a set of suitable decisions; we need to choose one
$\longrightarrow$ Model extraction



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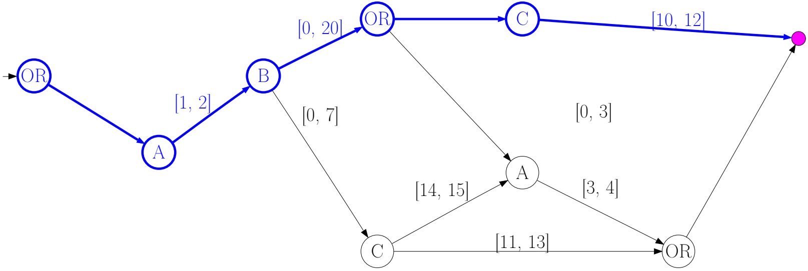

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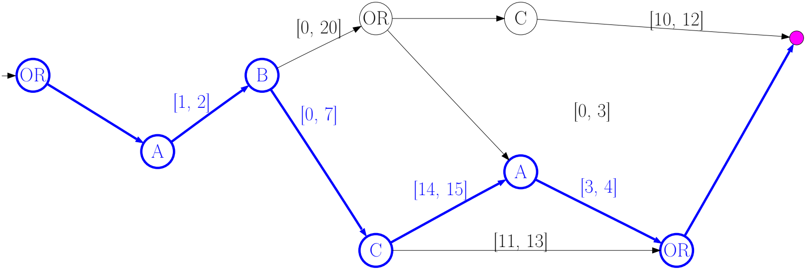

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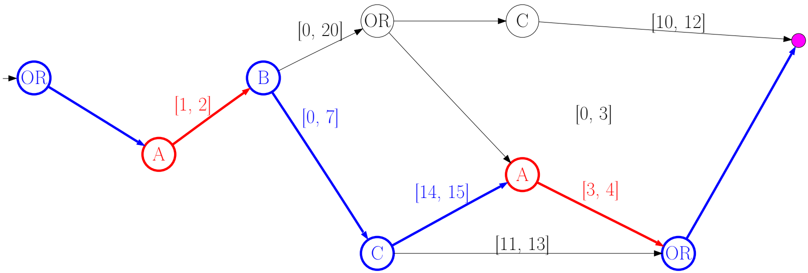


Model Extraction is Hard

- Paths in an IA can be inconsistent

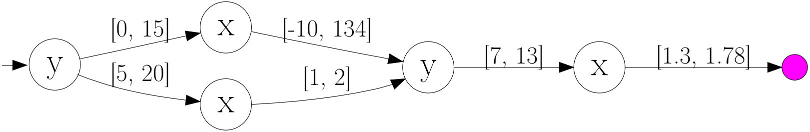
Theorem

Interval automata do not support model extraction in polytime.

$\rightarrow$ we look for a restriction on IAs, making this operation easier.

Focusing Interval Automata

- Idea: along a path, intervals can only shrink → focusing IAs



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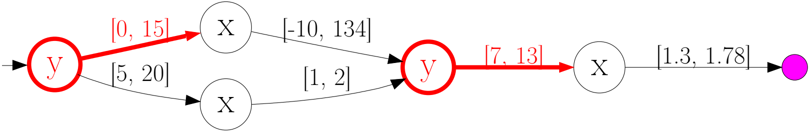

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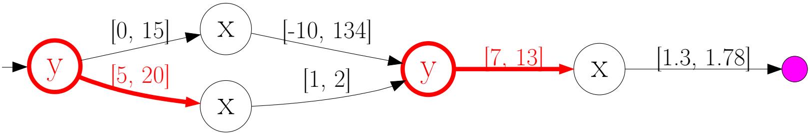

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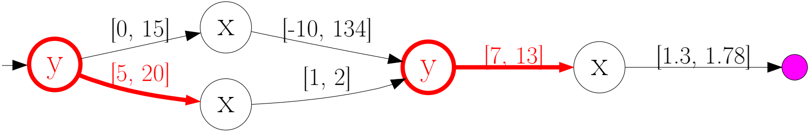


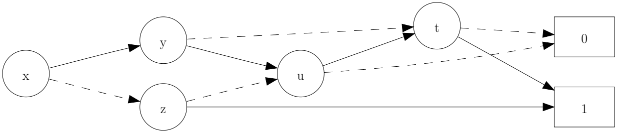
- Each path of a reduced FIA corresponds to at least one model

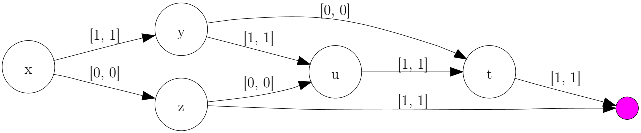
Theorem

FIAs support model extraction in polytime.

Read-once ⇒ Focusing

- Parallel with the "read-once" restriction on BDDs
- Read-once BDDs (FBDDs and OBDDs) are particular FIAs.



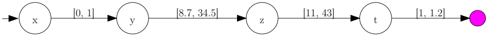


Outline

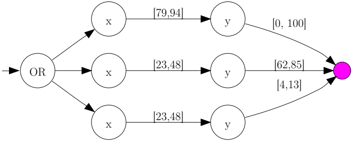
Compiling List of Boxes

- Compilation of decision policies, obtained with an external algorithm: list of "boxes" $\longmapsto$ FIA
  - Build the FIA representing each "box"

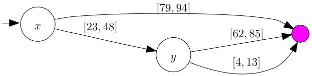
$$[0,1] \times [8.7, 34.5] \times [11, 43] \times [1, 1.2]$$



  - Make the disjunction of the boxes ($\vee$**C** easy on FIAs)



$\Rightarrow$

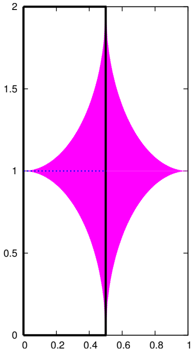


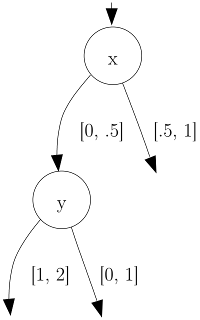
Compiling Continuous Constraint Networks

- Compilation of transition relations:

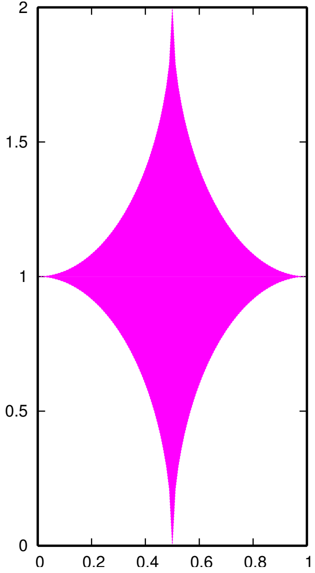
  continuous constraint networks $\longmapsto$ FIA

  - Following the "DPLL with a trace" approach: use the trace of the interval-based CSP solver RealPaver



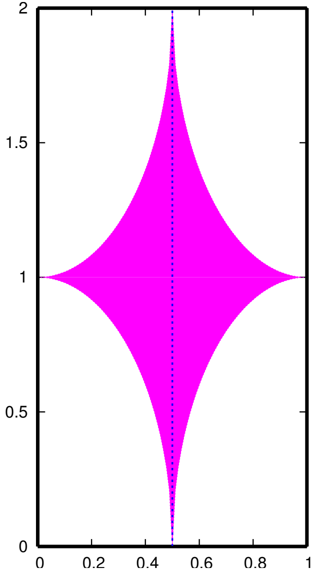


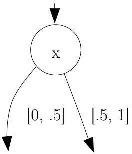
RealPaver with a Trace



$$\begin{cases} y \geqslant 2\sqrt{\max(0, 0.25 - x^2)} \\ y \leqslant 2 - 2\sqrt{\max(0, 0.25 - x^2)} \\ y \geqslant 2\sqrt{\max(0, 0.25 - (x-1)^2)} \\ y \leqslant 2 - 2\sqrt{\max(0, 0.25 - (x-1)^2)} \end{cases}$$

RealPaver with a Trace





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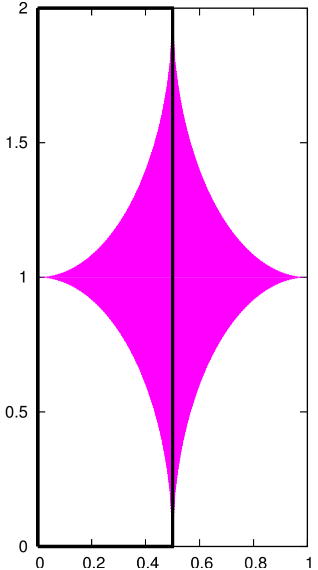

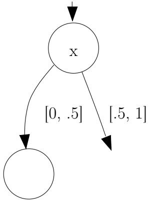

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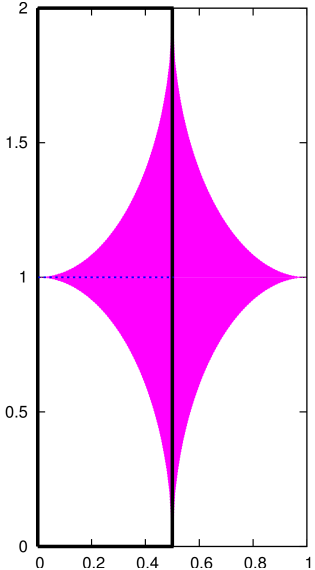

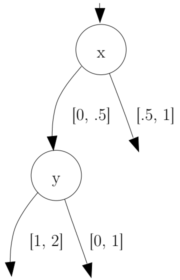

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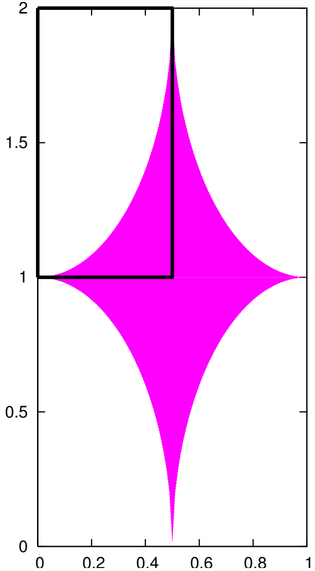

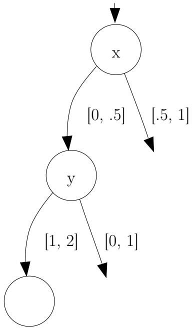

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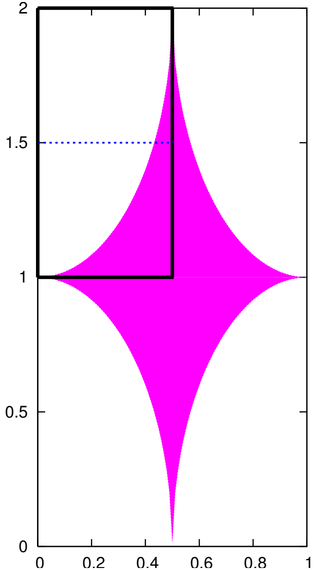

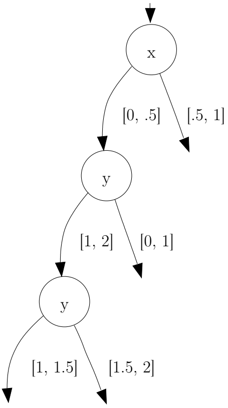

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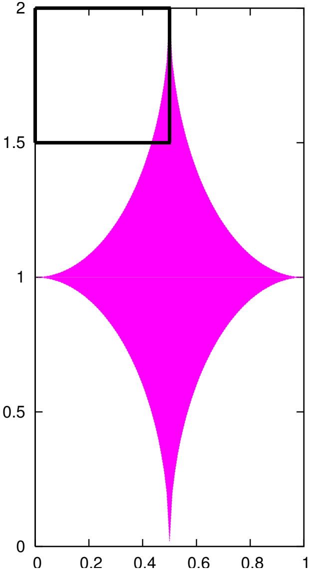

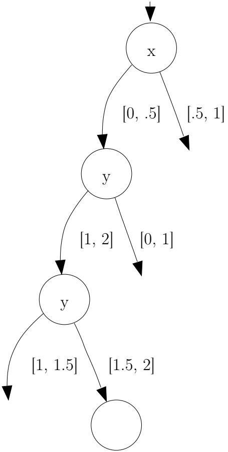

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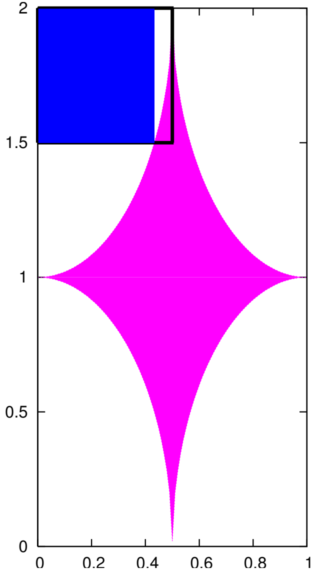

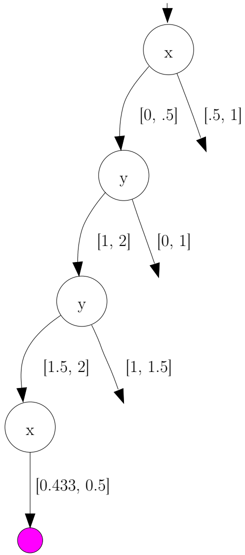

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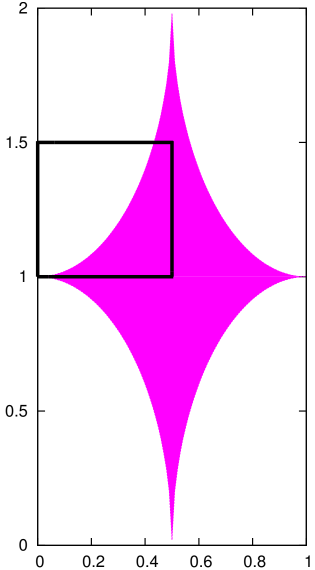

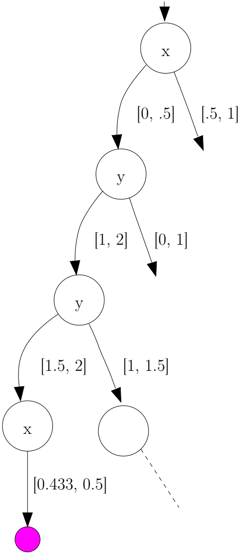

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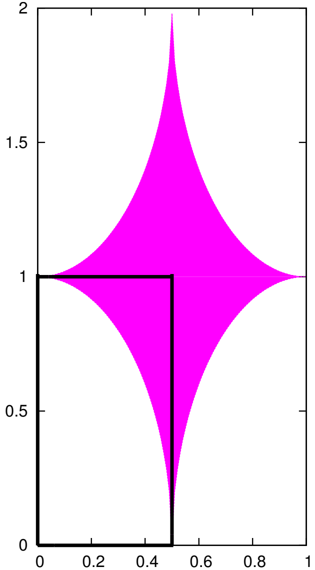
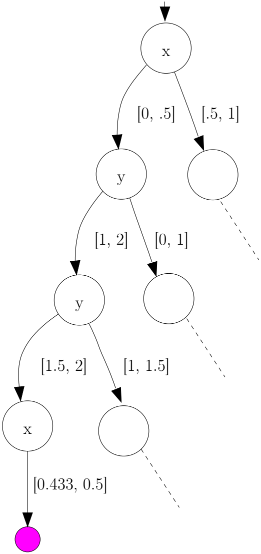




Implementation

- Prototype of "RealPaver with a trace"
- Toolbox for manipulating IAs/FIAs (**CO**, **MX**, **CD**, **FO**, $\wedge$**C**, ...)
- Experimenting exploitation of a transition table

problem	#edges	**CDFOMX**	**CDMX**	RealPaver
*Drone*4-5-3	61 596	<1 ms	1 ms	23 ms
*Drone*4-10-3	81 290	<1 ms	<1 ms	21 ms
*Drone*4-15-3	269 913	1 ms	1 ms	25 ms
*Drone*4-20-3	350 818	1 ms	1 ms	25 ms
*Drone*4-25-3	354 772	3 ms	3 ms	28 ms

$\rightarrow$ compatible with an online use

Knowledge Compilation Map of IAs and FIAs

Query	IA	FIA	DNNF
CO	∘	√	√
VA	∘	∘	∘
MC	√	√	√
CE	∘	√	√
IM	∘	∘	∘
EQ	∘	∘	∘
SE	∘	∘	∘
MX	∘	√	√
CX	∘	√	√
CT	∘	∘	∘
ME	∘	√	√

Transfo.	IA	FIA	DNNF
CD	√	√	√
TR	∘	√	√
FO	∘	√	√
SFO	√	√	√
EN	∘	∘	∘
SEN	√	∘	∘
∨C	√	√	√
∨BC	√	√	√
∨clC	√	√	√
∧C	√	∘	∘
∧BC	√	∘	∘
∧tC	√	√	√

√ polytime
∘ not polytime unless P = NP
• not polytime

Back to Enumerated Variables

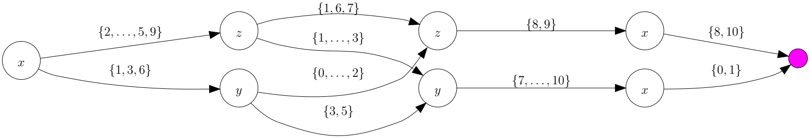
- Focusing IAs are not decomposable
- Yet the operations they support are similar to DNNF

→ We study how we can apply focusingness to languages on discrete variables

Outline

Set-labeled Diagrams

- We described a discrete counterpart to IAs: set-labeled diagrams (SDs)



- Edges of IAs labeled by intervals of real numbers
  $\rightarrow$ Edges of SDs labeled by sets of integers

The SD Family

- We considered a number of sublanguages of SDs, based on the following properties:
  - focusingness

    $\rightarrow$ FSDs
  - exclusive decision: imposes sets of sister edges to be disjoint

    $\rightarrow$ SDDs (set-labeled decision diagrams), FSDDs
  - fixed order on variables

    $\rightarrow$ OSDs and OSDDs

Relationship with Other Languages

- MDDs are particular OSDDs (all sets are singletons)

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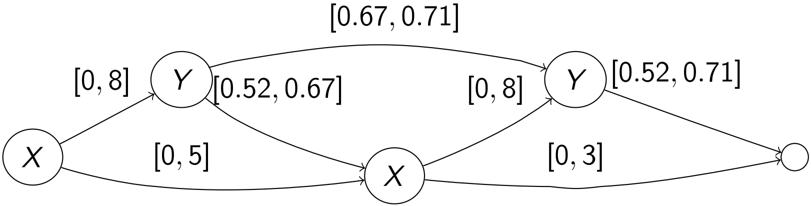
- FIAs can be "further compiled" into FSDs: judicious discretization



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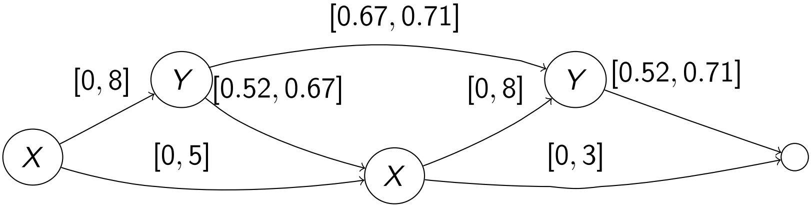
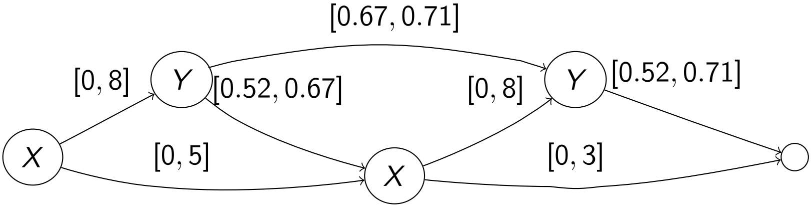


$$\{[0,3],\quad]3,5],\quad]5,8],\quad]8,10]\quad \}$$

$$\{[0,0.52[,\quad [0.52,0.67[,\quad \{0.67\},\quad]0.67,0.71]\quad]0.71,1]\quad \}$$

Relationship with Other Languages

- MDDs are particular OSDDs (all sets are singletons)
- FIAs can be "further compiled" into FSDs: judicious discretization



$$\{[0,3],\quad]3,5],\quad]5,8],\quad]8,10]\quad\}$$
$$\{x_1,\quad x_2,\quad x_3,\quad x_4\quad\}$$

$$\{[0,0.52[,\quad [0.52,0.67[,\quad \{0.67\},\quad]0.67,0.71]\quad]0.71,1]\quad\}$$

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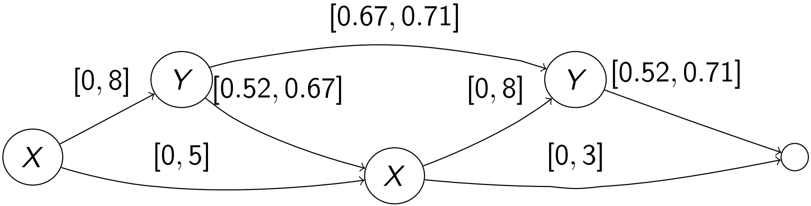

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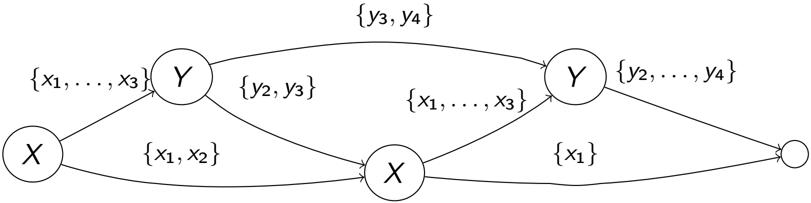

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KC Map of the SD Family: Queries

Query	SD	SDD	FSD	FSDD	OSD	OSDD	$\text{OSD}_<$	$\text{OSDD}_<$	DNNF
CO	$\circ$	$\circ$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
VA	$\circ$	$\circ$	$\circ$	$\surd$	$\circ$	$\surd$	$\circ$	$\surd$	$\circ$
MC	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
CE	$\circ$	$\circ$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
IM	$\circ$	$\circ$	$\circ$	$\surd$	$\circ$	$\surd$	$\circ$	$\surd$	$\circ$
EQ	$\circ$	$\circ$	$\circ$	?	$\circ$	$\surd$	$\circ$	$\surd$	$\circ$
SE	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\surd$	$\circ$
MX	$\circ$	$\circ$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
CX	$\circ$	$\circ$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
CT	$\circ$	$\circ$	$\circ$	?	$\circ$	$\surd$	$\circ$	$\surd$	$\circ$
ME	$\circ$	$\circ$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$

$\surd$ polytime
$\circ$ not polytime unless $P = NP$
$\bullet$ not polytime

KC Map of the SD Family: Transformations

Transfo.	SD	SDD	FSD	FSDD	OSD	OSDD	$\text{OSD}_<$	$\text{OSDD}_<$	DNNF
CD	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
TR	$\circ$	$\circ$	$\surd$	$\circ$	$\surd$	$\bullet$	$\surd$	$\bullet$	$\surd$
FO	$\circ$	$\circ$	$\surd$	$\circ$	$\surd$	$\bullet$	$\surd$	$\bullet$	$\surd$
SFO	$\surd$	$\surd$	$\surd$	$\circ$	$\surd$	$\bullet$	$\surd$	$\bullet$	$\surd$
EN	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\bullet$	$\circ$	$\bullet$	$\circ$
SEN	$\surd$	$\surd$	$\circ$	$\circ$	$\circ$	$\bullet$	$\circ$	$\bullet$	$\circ$
$\lor$**C**	$\surd$	$\surd$	$\surd$	$\circ$	?	$\bullet$	$\surd$	$\bullet$	$\surd$
$\lor$**BC**	$\surd$	$\surd$	$\surd$	$\circ$	?	$\circ$	$\surd$	$\surd$	$\surd$
$\lor$**clC**	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
$\land$**C**	$\surd$	$\surd$	$\circ$	$\circ$	$\circ$	$\bullet$	$\circ$	$\bullet$	$\circ$
$\land$**BC**	$\surd$	$\surd$	$\circ$	$\circ$	$\circ$	$\circ$	$\surd$	$\surd$	$\circ$
$\land$**tC**	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
$\neg$**C**	?	$\surd$	$\circ$	?	$\circ$	$\surd$	$\circ$	$\surd$	$\circ$

$\surd$ polytime
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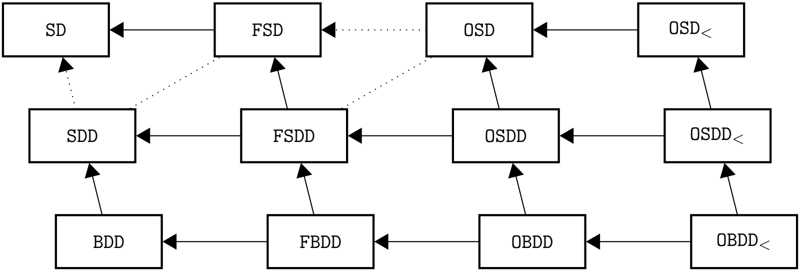
KC Map of the SD Family: Succinctness



- $\mathrm{L}_1 \longleftarrow \mathrm{L}_2$: L_1 is strictly more succinct than L_2
- $\mathrm{L}_1 \dashleftarrow \mathrm{L}_2$: unknown relationship

Choco with a Trace

- To compile discrete CSPs into FSDDs: "Choco with a trace"
- Different settings of Choco → different sublanguages
  - if variables are examined in fixed order → OSDDs
  - to get "pure" FSDDs, we can design heuristics for dynamic variable choice

Outline

Contributions

- Definition of interval automata and set-labeled diagrams
- Identification of the focusingness property
- KC map of IAs and FIAs / of the SD family
- "RealPaver/Choco with a trace" algorithms
- Implementation (compilers and toolboxes)

Perspectives

- Approximate compilation
- AND-nodes
- compilation modulo theory: more expressive literals
  $x \in [1, 4] \quad \longrightarrow \quad x - y < 3$
- Other data structures as compiled forms:
  $R^\star$-trees, hashtables...
- Study comparison of languages w.r.t. a representation change